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SCIENCE, GEOGRAPHY, AND MATHEMATICS.[1]

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THE time is short, and I shall have to be as brief as possible in my remarks.

There are a few suggestions that come to my mind which possibly may be interesting, perhaps helpful, to some of you in regard to the teaching of arithmetic.

I think we often make a mistake in teaching arithmetic, in not distinguishing between the kinds of processes that we are dealing with, and it may be profitable to discuss for a moment what some of these processes are.

There are, in the first place, the number-processes, and many people assume that these are the only ones that we deal with in arithmetic; but arithmetic is broader than number. Let us see what these processes are. They are the four fundamental "rules," as we call them — square root, cube root, and arithmetical and geometrical series; not all of which we should teach in the elementary schools. I cannot this moment think of any other mathematical process in arithmetic, unless you regard the application of the four fundamental rules to fractions as new mathematical processes, as I would not.

Besides these number-processes in arithmetic there is the language of numbers to be taught. All figure work is language work except in so far as it involves the number-processes. The teaching of figures, then, is language work. The Arabic notation is an arbitrary and technical language, very useful indeed, and is an effective means of handling numbers; but nevertheless it is language. Now, as a pedagogical point, we must bear in mind that the methods of teaching language are quite different from the methods of developing a thought-process. You develop a thought-process by leading the pupil to see the facts, and then to analyze them and draw his own conclusions. It is a process of development pure and simple, and in the development of that thought-process good pedagogy requires that we do not tell the pupil anything, but lead him to see the facts, to analyze them, and to draw his own conclusions.

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[1] Discussion of President DAVID FELMLEY'S paper on "The Teaching of Mathematics," at the dedication exercises of the School of Education, The University of Chicago, May 13, 1904.

When you come to language you deal with an arbitrary thing. That you call a cat a cat, and not a dog or something else, is purely arbitrary. To you and me words are not purely arbitrary, because we know the rudiments of philology; but to children they are absolutely so.

When you deal with figures you deal with language, *i. e.*, with the arbitrary. The figure 5 is a purely arbitrary symbol to the child. The Romans used one kind of symbol; the Hindus another, which we have adopted; the Greeks another; and besides these the words for five in the various languages may be used to represent the same thing. A great many teachers imagine that these symbols are actual numbers; that the figure 5, for example, is the abstract number five. They mistake figures for abstract numbers. If the figure 5 is the abstract number five, the word "five" is the abstract number five also, and the word "cat" is an abstract of cat. Abstract ideas cannot be thought except in connection with words, but words are not identical with abstract ideas.

While a thought-process must be developed, and nothing must be told to the pupil except facts where, as in history, he cannot actually observe them, in teaching language you must absolutely tell all, because all is arbitrary; so that at the beginning the process is a question of information, next a question of association of symbol with idea, and thirdly a question of drill. When you have these three things combined and have enlisted the pupil's interest, you have good language-teaching.

These characteristics of good language-teaching are not the characteristics of teaching which aims to develop a thought-process. We must distinguish very sharply between the two. Even the best teachers I know occasionally go wrong in this respect, and teach thought-processes in arithmetic as if they were language lessons.

What part of arithmetic, then, is language work and ought to be taught by the methods appropriate to the teaching of language, in distinction from those to be employed in the development of a mathematical thought-process?

In the first place, the Arabic notation is to be taught as language. In the second place, all the figure work in fractions. A fraction is a part of a thing, and our rules for fractions are not based on real fractions, but on the technical language of fractions. A real fraction, like half a bushel of apples or half an hour, has neither a numerator nor a denominator. These distinctions, on which our rules are based, are applied only to the technical language of fractions. If we should discard the Arabic notation and adopt the Roman, all our rules for fractions would become inapplicable. These rules are of great practical value, but we must recognize the fact that they are based on one arithmetical notation.

In the third place, when you teach decimals you are teaching, of course, simply a certain type of vulgar fractions having multiples of ten for a denominator, so there is absolutely nothing new there but the language. Now, what does that mean, pedagogically? It means that when you come to decimals you are teaching a language lesson, and you ought to teach it by the method by which you would teach French or German, or any other language, and not emphasize the mathematical processes involved, because these the child already knows. It is a question of simply telling him how a decimal is formed, showing him the identity between decimal and common fractions, making the association firm and strong between the decimal language and the ideas, and drilling upon it until he can think readily in this new technical language.

Not to go any farther into details, all technical terms in arithmetic are language. There is, therefore, a large part of arithmetic which is nothing but language, that must be taught by the method of teaching language. When you teach a language lesson in arithmetic like decimals as if it involved new number-processes, you attempt to teach the child what he knows by means of a language which he does not know. The emphasis of the teaching must be laid on the new language, else there is waste.

To pass on: Are there any other topics in arithmetic differing essentially in character from number-processes on the one hand, and from language on the other? Take a problem like this: "If one man can do a piece of work in three days, and another man in four days, how long will it take both of them to do it, working together?" What is the function of such a problem? It seems to me its function is to train the child to see conditions. A similar problem is that of the two boats: "Two boats start at the same place; one sails up the river, another down the river; the current is so many miles an hour; how far apart will they be at the end of so many hours?" What is the function of such a problem? It is to train the mind to picture conditions and reason upon them. Unless you see one boat go up and the other down the river, and keep in mind the velocity of the current, you cannot possibly solve it. The mathematical processes involved are simply the four fundamental rules already familiar to the pupil. Now, unless you teach such a problem by actually showing the conditions, you miss the point to be taught. If you try to work toward a rule and to show that you add the results when the boats go in opposite directions, and subtract if they go in the same direction, you miss all the value of such a problem. There is therefore a topic—formerly called "analysis"—which presents neither new number-processes nor new technical language, whose function it is to develop the power of analysis, and that is one of the important functions of arithmetic. What is the new element which is involved in the various topics which come under what we call "commercial arithmetic"? Take "interest," for example—what new element is involved upon which the teaching must lay the emphasis? Not a mathematical process, because the subject involves only the four fundamental rules. There is a little technical language which must be made familiar to the pupil. What is the new element to be taught? It is the nature of the business transaction which makes the paying of interest a profitable thing. The aim should be to lead the class to imagine circumstances or conditions under which one of them could borrow $100 or $200, keep it for a year or so, and then pay back the sum with 6 per cent. interest and yet make money.

When we take bank discount, what is there specially to be taught? Not mathematics; for there are only the four fundamental rules involved in it. What is the thing to be taught? It is the business operation—the function of a bank as a money-lending institution, and the method of calculating the interest on a loan. Problems should be solved, not for the purpose of securing expertness, but for the purpose of making clear the nature of the business transaction involved in borrowing money at a bank.

When you come to "stocks and bonds," what is the new element to be taught? Again there is no number or mathematical process involved which the children do not know. These topics present a little technical language that is unfamiliar and must be taught, but the burden of the teaching must be to make clear to the class the conditions in industrial and commercial life which make stocks and bonds necessary. It means that the general nature of corporations must be made clear to the class, the nature of stocks and of bonds, the difference between dividends and interest on bonds, the relative safety of stocks and bonds as investments, and the relation between low rate of interest and safety of investment. The fundamental ideas involved in all this can readily be grasped by grammar-school pupils of the upper grades. Little emphasis should be laid on the solving of problems except for the purposes of making clear and impressing the instruction on these various subtopics. "Stocks and bonds" taught in this way have a high practical value for every child; taught as topics in mathematics, with the emphasis on rules and the solution of problems, they are not worthy a place in the curriculum. There is a moral value in teaching these topics as suggested. If in the grammar schools of this country, in the two higher grades, we should impress upon the minds of children the relative degree of safety of stocks and bonds, by making them see clearly the nature of both and their difference, and if we should impress upon them the facts that the higher the rate of interest on bonds, the less safe they are, and that there is also some relation between dividends on stocks and safety; that there are plenty of millionaires throughout the country who are buying up all the stocks and all the bonds which pay a high rate of interest, and are at the same time safe, and that therefore those which pay a high rate of interest and which are glowingly advertised for sale are not safe investments, could the mining sharks in the West and in Boston and New York have fleeced the teachers and working people, as they have within the last two or three years, throughout the whole country, by selling them millions of dollars of worthless stocks?